Empirical Risk in Terms of ROC variables

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1 Abstract

Two commonly used performance metrics for two class classification algorithms are the receiver operating characteristic (ROC) curve and Empirical Risk. These two performance metrics can be related in order to directly compare scores once a point on an ROC curve is known. ROC is a more general description overall, but with certain assumptions we can compute one in terms of the other.

2 ROC and Empirical Risk

The ROC variables TPF and FPF, (the true positive fraction and false positive fraction respectively) are defined as

$$FPF = \lim_{N\to\infty} \frac{N_{\text{fp}}}{N_{\text{fp}} + N_{\text{tn}}} \approx \frac{N_{\text{fp}}}{N_{\text{fp}} + N_{\text{tn}}}, \tag{2.1}$$

$$TPF = \lim_{N\to\infty} \frac{N_{\text{tp}}}{N_{\text{tp}} + N_{\text{fn}}} \approx \frac{N_{\text{tp}}}{N_{\text{tp}} + N_{\text{fn}}} \tag{2.2}$$

where N_{tp} is the number of true positives, N_{fp} is the number of false positives, N_{tn} is the number of true negatives, and N_{fn} is the number of false negatives [1]. Note that here, we have dropped any functional notation referring to the specific classifier observer itself. The various Ns are all *outputs of some generic classifier*. The empirical risk is defined as :

$$R_{\text{emp}} = \frac{N_{\text{fn}} + N_{\text{fp}}}{N} \tag{2.3}$$

where

$$N = N_{\text{fn}} + N_{\text{tn}} + N_{\text{fp}} + N_{\text{fp}} \tag{2.4}$$

is the total number of observations. This relationship can be deduced from the Empirical Risk equation in Vapnik [2]. When R_{emp} is a valid metric, the target and background/clutter must be approximately 50% each of the data, this implies that

$$N_{\text{fp}} + N_{\text{tp}} \approx N_{\text{fn}} + N_{\text{tn}} \approx \frac{1}{2}N \tag{2.5}$$

After some substitutions we can arrive at :

$$R_{\text{emp}} = \frac{1}{N}\left(\frac{FPF}{1-FPF}N_{\text{tn}} + \frac{1-TPF}{TPF}N_{\text{tp}}\right) \tag{2.6}$$

$$= \frac{2N_{\text{tn}}}{N_{\text{tn}} + N_{\text{fn}}}\frac{FPF}{1-FPF} + \frac{2N_{\text{tp}}}{N_{\text{tp}} + N_{\text{fp}}}\frac{1-TPF}{TPF} \tag{2.7}$$

References

[1] Harrison H. Barrett and Kyle J. Myers. *Foundations of Image Science.* Wiley-Interscience, Hoboken, New Jersey, United States, 2004.

[2] Vladimir Naumovich Vapnik. *The Nature of Statistical Learning Theory.* Springer Verlag, New York, 2000.